

**2015 John O'Bryan Mathematical Competition
Questions for the Two-Person Speed Event**

*****Calculators may not be used on the first four questions*****

- Let $A = \{1, 2, 3, 4\}$. Let k be the number of *distinct* subsets of A that contain the element 3. Let w be the number of distinct positive integral factors of 2015. Find the product (kw) .
- Let $\begin{vmatrix} 2w & 100 \\ 50 & 10 \end{vmatrix} = 2200$. Let k be the probability that the sum of the two top faces is six when randomly throwing two fair 6-sided dice (the faces of each die are labeled with the values 1, 2, 3, 4, 5, 6). Find the product (kw) .
- Two sequences are defined as follows: $\begin{cases} a_1 = 2 \\ a_n = a_{(n-1)} + 3 \end{cases}$ and $\begin{cases} b_1 = 3 \\ b_n = b_{(n-1)} + 3.5 \end{cases}$. Find the arithmetic mean of the first three common terms of these sequences.
- Let $y = mx + b$ be the equation of a line that contains $(2, 18)$ and is perpendicular to the line whose equation is $x + 5y = 10$. Let k be the smallest positive improper fraction that each of the following will divide exactly, resulting in integral quotients: $\frac{3}{13}, \frac{5}{26}, \frac{7}{39}$. Find $k(m + b)$.

*****Calculators may be used on the remaining questions*****

- Let n be the smallest possible positive integer such that there are at least 2900 distinct committees possible when appointing a 3-member committee from n persons. Let r be the length of the radius of the inscribed circle of a triangle whose side lengths are 13, 37, and 40. Find the value of the product (nr) .
- Find the sum of all distinct values of x such that the three terms $x + 7$, $5x - 6$, and $7x + 2$ taken in **some order** form an arithmetic sequence. Express your answer as an improper fraction reduced to lowest terms.
- Determine the positive difference between the number of digits in 83^{2015} and the units digit of 83^{2015} .
- Let $ABCDEF$ be a regular hexagon. Let $k = \frac{m\angle FAB}{m\angle AFB + m\angle ABF}$. Let w be the number of gallons of water that should be evaporated from 120 gallons of a solution that is 30% salt to obtain a solution that is 50% salt. Find the value of $(k + w)$.
- (T1) A company produces golf balls only in boxes of 5 and 12. Thus if you wanted to purchase 14 balls, you could not get 14 exactly with any combination of whole boxes of balls. Find the largest number of balls you could not get exactly with some combination of whole boxes of balls.
- (T2) Find the value of $10111_{\text{two}} + 2202_{\text{three}} + 3212_{\text{four}} + 544_{\text{six}}$. Express your answer in **base ten**.

Names: _____

Team Code: _____

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Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value; however ties for individual awards will be broken based on problem difficulty.

SCORE

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

T1. _____

T2. _____

Calculators are not allowed to be used on the first four questions!

This competition consists of eight competitive rounds. Correct answers will receive the following scores:

1st: 7 points
2nd: 5 points
All Others: 3 points

There is a three minute time limit on each round. You may submit only one answer each round. To submit your answer, fold this sheet **lengthwise** and hold it high in the air so that a proctor may check your answer.

TOTAL SCORE

Name: _____ **ANSWERS** _____

Team Code: _____

**2014 John O'Bryan Mathematical Competition
Answers for the Two-Person Speed Event**

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value; however ties for individual awards will be broken based on problem difficulty.

1. 64

2. 50

3. 38

4. 105

5. 144

6. 477 / 40 Must be this fraction

7. 3860

8. 50

T1. 43

T2. 535

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